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A direct method for evaluating performance of horizontal axis wind turbines

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Abstract

This paper presents a direct approach for the determination of aerodynamic performance characteristics of horizontal axis wind turbines. Based on Glauert's solution of an ideal windmill along with an exact trigonometric function method, analytical closed form equations are derived and given for preliminary determination of the optimum chord and twist distributions. The variation of the angle of attack of the relative wind along blade span is then obtained directly from a unique equation for a known rotor size and refined blade geometry. A case study including the analysis of an existing turbine model is given and results are discussed and compared with those obtained by other investigators. It is shown that the approach used in this study is efficient and saves much of the computational time as compared with the commonly used iterative procedures. © 2001 Elsevier Science Ltd. All rights reserved.

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Nomenclature

a	Axial induction factor
a'	Angular induction factor
A_D	Effective disk area swept by the rotating blades
C	Blade chord
C_n	Normal force coefficient
C_t	Tangential force coefficient
C_D	Drag coefficient
C_L	Lift coefficient
C_P	Power coefficient
C_Q	Torque coefficient
C_T	Thrust coefficient
F	Tip-loss factor
N_B	Number of blades
P	Extracted power
Q_{aero}	Aerodynamic torque
r	Local blade radius
R	Rotor radius
T_{aero}	Aerodynamic thrust
V_o	Mean wind velocity at hub height
V_r	Resultant wind velocity
V_w	Net wind velocity
α	Angle of attack
θ_B	Airfoil setting angle
θ_o	Initial built-in twist
θ_s	Setting angle at blade root (Blade pitch)
λ	Tip-speed ratio
λ_r	Local speed ratio
ρ	Air density
σ	Local solidity
ϕ	Inflow angle

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1. Introduction

The success of wind power as an alternate source of energy is a direct function of the economics of designing and manufacturing the wind machine. One of the most important aspects of the design is the development of performance prediction methods. It is the purpose of this study to present analytical and direct procedures for evaluating performance characteristics of horizontal-axis wind turbines. Glauert [1,2] originated the basic aerodynamic analysis concepts of airscrew propellers and windmills. He applied first the momentum and energy relationships for simple axial flow and then considered the effects of flow rotation after passing through the rotor as well as the secondary flows near the tip and hub regions. Wilson et al. [3,4] extended Glauert's work and presented a step by step procedure for calculating performance characteristics of wind turbines. Analysis was based on a two-dimensional blade-element strip theory and iterative solutions were obtained for the axial and rotational induction factors. Later on, Wilson [5] outlined a brief review of the aerodynamics of horizontal-axis wind turbines. The performance limits were presented, and a short note was given on the applicability of using the vortex-flow model.

This paper focuses on the improvement of the methods of performance analysis. The suggested approach is divided into two convenient stages, namely; (a) the evaluation of a preliminary blade geometry, i.e. the chord and twist distributions, and (b) the calculations of the various aerodynamic coefficients for a refined blade geometry which can be practical from the manufacturing and cost points of view. In the first stage a closed-form cubic equation in the axial induction factor is developed according to Glauert's optimum solution of an ideal windmill [1]. This is then solved analytically by an exact trigonometric function method yielding the optimum chord and twist distributions. The optimum working operating condition of the airfoil section is considered which leads to the maximum lift-to-drag ratio. Based on strip theory equations [4], the second stage derives a unique equation in the angle of attack of the resultant wind velocity and the various aerodynamic coefficients are then determined systematically. The method is investigated by testing an existing turbine model and the results are compared with those obtained by other investigators. It is shown that the proposed approach simplifies and reduces much of the computational efforts as needed by the traditional iterative methods.

2. Performance-optimized wind turbine

A wind turbine system design should be based on the cost-effective production of energy, which depends very much on the site wind characteristics and machine

performance characteristics. Maximization of the annual energy production may be attained by maximization of the rotor power coefficient. This depends, for a prescribed tip-speed ratio, on the type of airfoil, chord and twist distributions and number of blades.

Several theoretical studies have been published for the determination of a performance-optimized wind turbine. Glauert [1] initiated the calculation of the optimum wind turbine by making the power integral equation stationary. He considered an ideal actuator disk model and obtained the optimum variations of the axial and rotational induction factors (a and a'), as well as the inflow angle, ϕ, as follows (refer to eqs. 2.10, 2.11 and 2.14, page 328 in Ref. [1]):

$$a'=\frac{1-3a}{4a-1} \tag{1a}$$

$$a'(1+a')\lambda_r^2=a(1-a) \tag{1b}$$

$$\tan\phi=\frac{(1-a)}{\lambda_r(1+a')} \tag{1c}$$

where $\lambda_r=\Omega r/V_w$ is the local speed ratio at the rth station along the blade, Ω is the rotational speed, and V_w is the net wind velocity. Fig. 1 shows a front view of a rotating blade and the velocity triangle of an arbitrary airfoil section. In deriving Eqs. (1a), (1b) and (1c), secondary effects of drag and tip losses were neglected. If one further neglects rotation behind the rotor (i.e. a=1/3 and a'=0), the inflow angle may be determined from the relation:

$$\phi=\tan^{-1}(2/3\lambda_r) \tag{2a}$$

which may be used only for $\lambda_r>1$.

Glauert presented the solutions of Eqs. (1a), (1b) and (1c) in a tabulated form (see

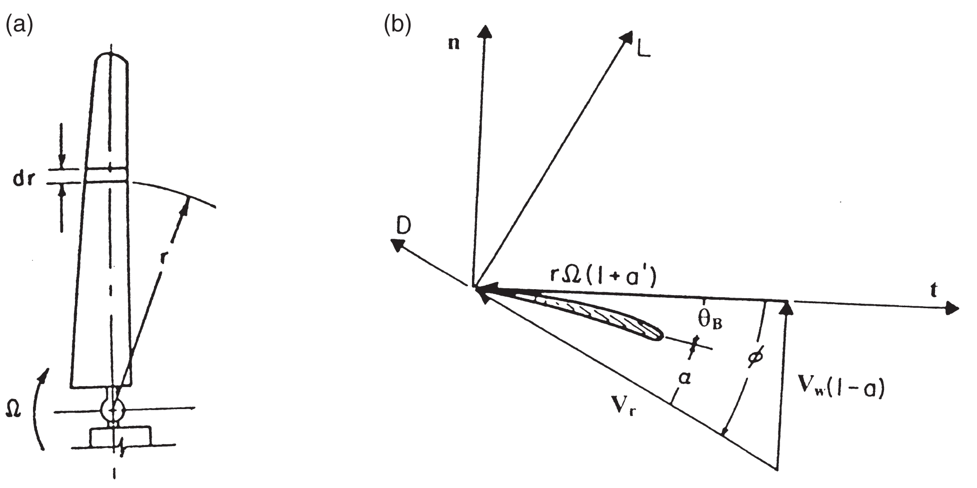

Fig. 1. Airfoil-blade geometry. (a) Front view of rotating blade; (b) Velocity triangle.

Table 34, Page 329 of Ref. [1]) by first assigning a value for the factor a, calculate the corresponding value of a', and then the local speed ratio λ_r. It is a major aim of the present study to calculate the factors a and a' directly from analytical expressions for a specified value of λ_r. Wilson et al. [4] performed a local optimization analysis by maximizing the power output at each radial station along the blade. The axial induction factor was varied until the power contribution became stationary (see page 59 of Ref. [4]). Rohrbach and Worobel [6] investigated the effect of blade number and section lift-to-drag ratio on the maximum turbine performance. Their results have been found to yield slightly lower maximum performance than that found in Ref. [4].

Optimum power coefficient was also investigated by Nathan [7] who derived an approximate relationship between the inflow angle (ϕ) and the local speed ratio (λ_r). The relation was given by the following 5th order polynomial:

$$\phi=57.51-35.56\lambda_r+10.61\lambda_r^2-1.586\lambda_r^3+0.114\lambda_r^4-0.00313\lambda_r^5 \quad (2b)$$

where ϕ is measured in degrees. Nathan's equation was obtained for a lift-to-drag ratio ranging from 28.6 to 66.6 and the effects of secondary flows in the tip and hub regions were not included in the analysis.

Fig. 2 shows the variation of the optimum inflow angle, ϕ, with the local speed ratio as calculated by the various methods. It is remarked that the results obtained by Wilson et al. (see Table 3.2.3, page 74 of Ref.[4]) are identical to those determined from Glauert's Eqs. (1a, 1b) and (1c). Deviations can only be significant at stations

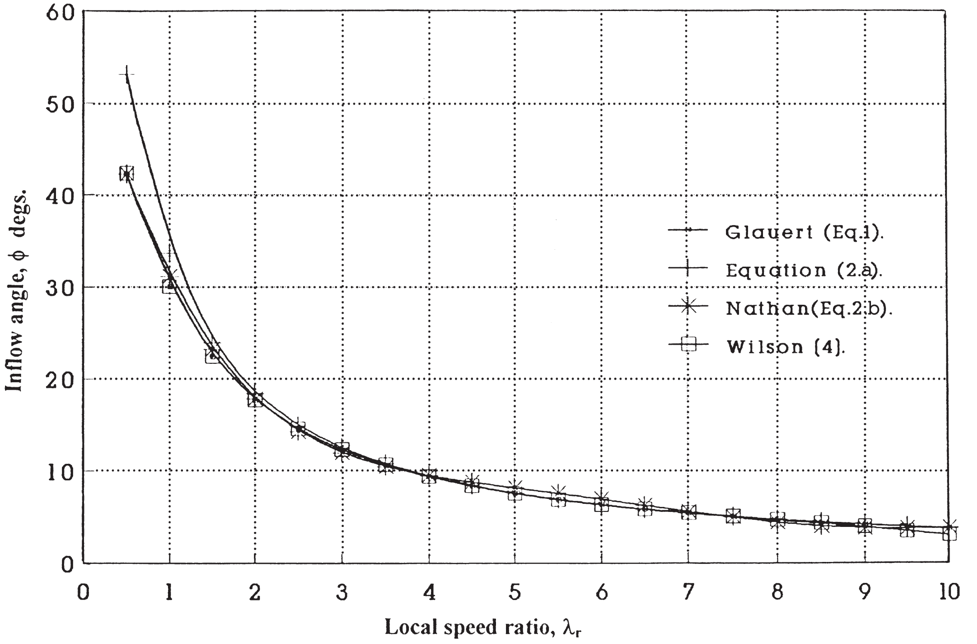

Fig. 2. Variation of optimum inflow angle with speed ratio.

very close to the blade tip region. Comparisons between the results obtained by Glauert [1] and Nathan [7] indicates that the major discrepancies occur near the blade root portion, while no significant difference is observed along the rest of the blade. More examination of Fig. 2 indicates that the optimum distribution of the inflow angle can be adequately determined from Glauert's solution. The next section will be devoted to the development of a closed-form analytic expression for the inflow angle as determined from Glauert's optimum conditions.

2.1. Analytical optimum Glauert's solution (an exact trigonometric function method)

Combining Eqs. (1a) and (1b), we get the following cubic polynomial in the axial induction factor a:

$$16a^3-24a^2+3(3-\lambda_r^2)a-(1-\lambda_r^2)=0 \tag{3}$$

Introducing the transformation $a=b\cos\theta+1/2$, and comparing with the trigonometric identity $4\cos^3\theta-3\cos\theta=\cos3\theta$, we obtain two values for the parameters b and θ as follows:

$$b^{\pm}=\pm\Lambda_r/2 \tag{4a}$$

$$\theta^{\pm}=\frac{1}{3}\cos^{-1}(\pm\Lambda_r^{-1}) \tag{4b}$$

$$\Lambda_r=\sqrt{1+\lambda_r^2} \tag{4c}$$

It can be shown that the root which ensure a positive value for the rotational induction factor must be given by the relation

$$a=\frac{1}{2}[1-\Lambda_r(\cos\theta^{+}-\cos\theta^{-})] \tag{5}$$

The factor a' and the corresponding optimum value of the inflow angle ϕ can then be determined automatically from Eqs. (1a) and (1c), respectively.

2.2. Optimum chord and twist distributions (preliminary blade design)

Glauert defined the theoretical aerodynamic efficiency of a blade element, η, as the ratio of the power absorbed by the element to the useful work done by the thrust on it (refer to page 214, eq. 2.3 of Ref. [1]). This leads to the relation:

$$\eta=\frac{1-\tan\gamma\cot\phi}{1+\tan\gamma\tan\phi} \tag{6a}$$

$$\tan\gamma = \frac{C_D}{C_L} \tag{6b}$$

It can be seen that the efficiency increases as the drag-to-lift ratio decreases. Fig. 3(a) shows the polar diagram of a typical airfoil section. It is shown that tanγ is minimum for the angle of attack, α, corresponding to the point where the straight

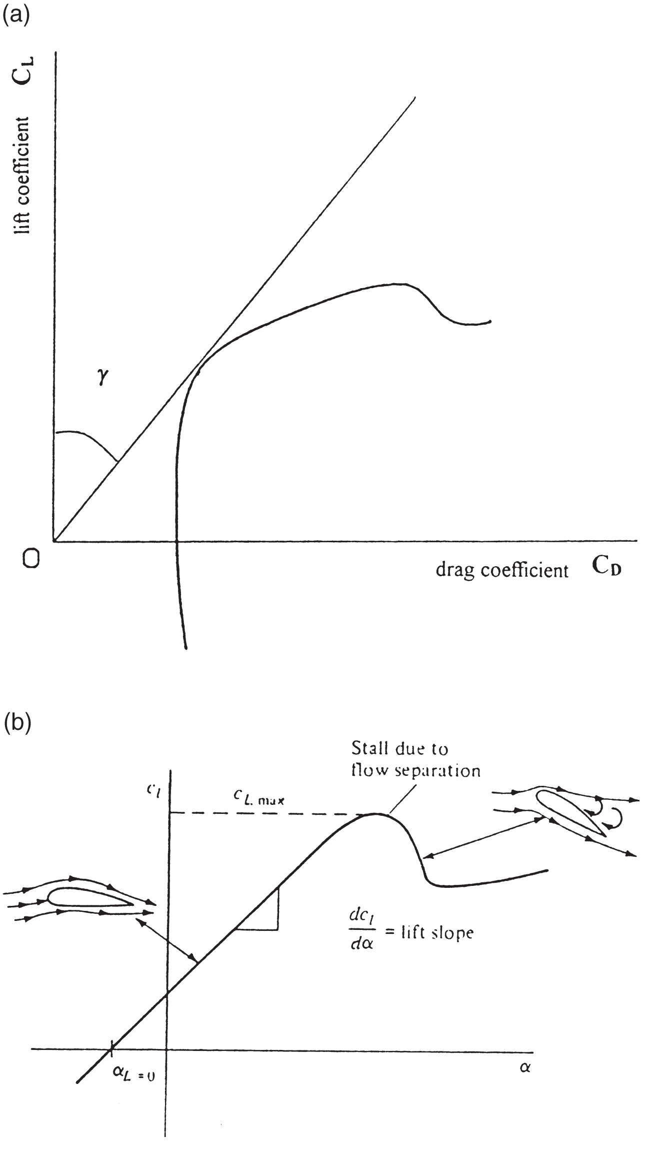

Fig. 3. Aerodynamic characteristics of a typical airfoil section. (a) Polar diagram; (b) $C_L-\alpha$ curve.

line from the origin becomes tangent to the curve. For this particular value of α, the efficiency reaches a maximum value and, hence, the turbine performance. For unstalled condition, the relation between C_D and C_L may be approximated by a second-degree polynomial in the form

$$C_D = C_{DO} + K_1 C_L + K_2 C_L^2 \tag{7a}$$

where C_{DO} is the drag coefficient at zero lift, and K_1 and K_2 are constant coefficients to be determined from the prescribed data of the airfoil section such as those presented in Ref. [8].

Therefore the optimum condition can be given by

$$C_L = \sqrt{C_{DO}/K_2} \tag{7b}$$

$$C_D = 2C_{DO} + K_1 C_L \tag{7c}$$

The corresponding minimum value of $\tan\gamma$ is equal to $K_1 + 2\sqrt{K_2 C_{DO}}$. The optimum angle of attack can then be determined from the relation

$$\alpha = \frac{C_L}{\partial C_L d/\partial\alpha} + \alpha_o \tag{8}$$

where α_o is the zero-lift angle of attack and $\partial C_L/\partial\alpha$ is the $C_L - \alpha$ curve slope (see Fig. 3(b)).

Referring to Fig. 1, the optimum twist angle can be calculated from the relation

$$\theta_B = \phi - \alpha \tag{9}$$

and the optimum distribution of blade chord, c, for given number of blades (N_B), is determined from

$$\sigma_r = \frac{N_B c}{\pi r} \tag{10}$$

where σ_r is termed as the local solidity ratio which can be calculated from Eq. (15) for optimum condition (see next section). It is to be noticed that, in Eq. (10), the dimensionless chord c and radial distance r are defined by the notations $c \leftarrow c/2R$ and $r \leftarrow r/R$, respectively. This means, for example, that the dimensionless airfoil chord is equal to its dimensional value divided by twice the rotor radius (refer to Table 1).

Table 1
Definition of dimensionless quantities

Quantity	Notation	Non-dimensionalization
Radial distance of blade element from hub center	r	$r \leftarrow r/R$
Radial distance of element from blade root	x	$x \leftarrow x/L$
Blade root offset from hub center	r_H	$r_H \leftarrow r_H/R$
Blade length	L	$L \leftarrow L/R$
Blade chord	C	$C \leftarrow C/2R$
Wind Velocity	(V_o, V_w, V_r)	$V \leftarrow V/\Omega R$

3. Aerodynamic performance analysis via a direct method

3.1. Basic equations

Wind turbine performance is determined in terms of aerodynamic thrust, torque and power. The present analysis utilizes the basic equations of strip theory (see eq 2.15, p. 329, Ref. [1]. Also see eqs 2.4.5–6, p. 23–24 and eq 2.6, p. 39 of Ref. [4]), where the axial and rotational induction factors at any radial station, r, along the blade are given by

$$\frac{a}{1-a}=\frac{\sigma_r C_n}{4F\sin^2\phi}\Rightarrow a=\frac{1}{\frac{4F\sin^2\phi}{\sigma_r C_\mathrm{n}}+1} \tag{11a}$$

$$\frac{a'}{1+a'}=\frac{\sigma_r C_\mathrm{t}}{4F\sin\phi\cos\phi}\Rightarrow a'=\frac{1}{\frac{4F\sin\phi\cos\phi}{\sigma_r C_\mathrm{t}}-1} \tag{11b}$$

The size and rotational speed of the rotor as well as the distribution of the chord and twist are all assumed to be preassigned parameters. The blade geometry may be determined from refined optimum distributions as obtained in the previous Section 2.2.

C_n and C_t are known as the normal and tangential aerodynamic coefficients, respectively, and can be determined from (see Fig. 1)

$$C_\mathrm{n}=C_\mathrm{L}\cos\phi+C_\mathrm{D}\sin\phi \tag{12a}$$

$$C_\mathrm{t}=C_\mathrm{L}\sin\phi-C_\mathrm{D}\cos\phi \tag{12b}$$

F denotes Prandtle's tip-loss factor defined by the relations [1,4]:

$$F=\frac{2}{\pi}\cos^{-1}(e^{-f}) \tag{13a}$$

$$f=\frac{N_\mathrm{B}}{2}\frac{(1-r)}{r\sin\phi} \tag{13b}$$

According to Wilson [4], the tip-loss factor is introduced herein as a first order contribution, which is referred to as the standard method of analysis. Dividing Eq. (11b) by Eq. (11a) and substituting in Eq. (1c), yields to

$$\frac{C_\mathrm{t}}{C_\mathrm{n}}=\frac{a'}{a}\lambda_r \tag{14}$$

Combining Eqs. (11a) and (11b) with Eq. (14) gives the following transcendental equation:

$$4F\sin\phi(\cos\phi-\lambda_r\sin\phi)-\sigma_r(\lambda_r C_{\mathrm{n}}+C_{\mathrm{t}})=0 \tag{15}$$

which represents a unique relation that can be used directly to determine the angle of attack and, hence, the various aerodynamic coefficients at the specified blade station. Any suitable root-finding interactive-computer routine, such as that based upon Newton–Raphson or Bisection method, can be used to find the required solutions of the above equation.

3.2. Calculation of aerodynamic thrust, torque and power

Having determined the variation of the angle of attack and the corresponding aerodynamic coefficients (C_{L}, C_{D}, C_{n}, C_{t}) along the blade span, the total thrust, torque and power developed by the N_{B}-bladed rotor can be computed by summing up the contributions of the individual blades and integrating over one complete revolution. Firstly, we calculate the dimensionless velocity of the relative wind from the relation (refer to Fig. 1 and Table 1).

$$V_{\mathrm{r}}^2=V_{\mathrm{w}}^2[(1-a)^2+\lambda_r^2(1+a')^2] \tag{16}$$

Secondly, the thrust, torque and power coefficients are determined from

$$\text{Thrust}-\text{Coefficient } C_{\mathrm{T}}=\frac{2N_{\mathrm{B}}L\int_0^1 V_{\mathrm{r}}^2 C C_{\mathrm{n}}dx}{\pi V_{\mathrm{o}}^2(1-r_{\mathrm{H}}^2)} \tag{17a}$$

$$\text{Torque}-\text{Coefficient } C_{\mathrm{Q}}=\frac{2N_{\mathrm{B}}L\int_0^1 V_{\mathrm{r}}^2 C C_{\mathrm{t}}r\,dx}{\pi V_{\mathrm{o}}^2(1-r_{\mathrm{H}}^2)} \tag{17b}$$

$$\text{Power}-\text{Coefficient } C_{\mathrm{P}}=\lambda C_{\mathrm{Q}} \tag{17c}$$

where $\lambda=\Omega R/V_{\mathrm{o}}$ is the tip-speed ratio of the rotating blade (refer to Appendix A for the details of deriving Eqs. (17a), (17b) and (17c)). Finally, the aerodynamic thrust, torque and power can be determined from the relations:

$$\text{Thrust } T_{\mathrm{aero}}=\frac{1}{2}\rho V_{\mathrm{o}}^2 A_{\mathrm{D}} C_{\mathrm{T}} \tag{18a}$$

$$\text{Torque } Q_{\mathrm{aero}}=\frac{1}{2}\rho V_{\mathrm{o}}^2 A_{\mathrm{D}} R C_{\mathrm{Q}} \tag{18b}$$

$$\text{Extracted Power } P=\frac{1}{2}\rho V_{\mathrm{o}}^3 A_{\mathrm{D}} C_{\mathrm{P}}=\Omega Q_{\mathrm{aero}} \tag{18c}$$

where A_D is the effective disk area swept by the rotating blades and R is the rotor radius.

4. Application (a case study)

As a case study, the proposed method of analysis will be implemented on an experimental, 100 Kw horizontal axis wind turbine; namely, the ERDA–NASA MOD-0 model. The machine operates at a design rotational speed of 40 rpm while producing its rated power at 8 m/s wind velocity. The rotor has two blades with 38.1 meters diameter located downwind from the tower at 30 meters height above the ground. The blades are preconed at 7° and have NACA 230xx-series airfoils. More about the configuration description and technical data can be found in Ref. [9].

4.1. Optimum chord and twist distributions

Based on the analytic procedure outlined in Sections 2.1 and 2.2, the optimum chord and twist distributions of the turbine model described above have been determined and presented in Table 2. To compare the results with those obtained by Wilson

Table 2
Optimum chord and twist distributions for MOD-0 turbine (C_L=0.9, α=8°, $C_D \cong 0$)

Station	Optimum Chord (meter)		Optimum	Twist (degrees)
r	Wilson [4]	Present	Wilson [4]	Present
0.05	3.4653	3.4673	34.255	34.3289
0.10	3.5643	3.5749	21.995	22.0489
0.15	3.0264	3.0384	14.460	14.5050
0.20	2.5210	2.5321	9.710	9.7490
0.25	2.1281	2.1378	6.534	6.5679
0.30	1.8288	1.8374	4.290	4.3192
0.35	1.5978	1.6054	2.630	2.6560
0.40	1.4161	1.4226	1.357	1.3804
0.45	1.2698	1.2757	+0.351	0.3732
0.50	1.1500	1.1553	-0.462	-0.4413
0.55	1.0506	1.0549	-1.134	-1.1130
0.60	0.9665	0.9697	-1.699	-1.6760
0.65	0.8952	0.8961	-2.182	-2.1546
0.70	0.8330	0.8309	-2.613	-2.5663
0.75	0.7794	0.7708	-2.978	-2.9241
0.80	0.7312	0.7115	-3.325	-3.2380
0.85	0.6867	0.6460	-3.661	-3.5155
0.90	0.6422	0.5610	-4.023	-3.7625
0.95	0.5919	0.4265	-4.494	-3.9839
1.00	0.0000	0.0000	–	–
Power Coeff C_P	0.547	0.5312		

et al. [4], the effect of drag is ignored, i.e. C_D is set equal to zero in Eqs. (12a) and (12b). All sections composing the blade are assumed to have NACA 23018 profile operating near the optimum working condition where C_L=0.9 and α=8°. The constant coefficients of Eq. (7a) have been determined from the data presented in Ref. [8] for Reynold's number equal to 3.0×10^6. They are:

$$C_{DO}=0.0067985,\ K_1=-0.00105613,\ \text{and } K_2=0.0068344.$$

Other parameters used in generating the results are:

$C_L-\alpha$ Curve Slope: $\partial C_L/\partial\alpha$=5.73 per radian.
Zero-lift angle of attack: $\alpha_o=-1.1°$
Max. lift coefficient: C_{lmax}=1.4

Moreover, the present analysis neglects the effect of wind shear and tower shadow which means that the net wind velocity, V_w, at the various blade stations can be taken equal to the mean wind velocity at hub height, V_o.

Examining the results presented in Table 2, it is seen that the values obtained by Wilson et al. [4] are too close to those calculated by the present method of analysis. Deviations can only be significant in the neighborhood of blade tip portion. It is also noticed that the difference in calculating the power coefficient (Eqs. (17b) and (17c)) is about 2.8%. The actual design case with $C_L/C_D\cong79$ has also been investigated where the power coefficient was found to be 0.4657 which is less than the value of 0.473 given in Ref. [4] by about 1.5%. Such discrepancies may be attributed to the fact that Wilson procedure contained a second-order contribution from the tip-loss factor. However, experimental results for wind turbines are not sufficiently accurate to ascertain the validity of including the 2nd order term.

4.2. Performance of a trimmed rotor (constant power condition)

For a preassigned blade configuration and rotor speed the power output of a wind turbine is basically a function of the wind velocity and pitch angle at blade root. Wind turbines are usually designed to generate, within their operating range, a constant amount of power at a specified rotor speed. Therefore, given the mean wind velocity at hub height, V_o, the problem is to determine the proper value of the pitch angle, θ_s , at root required to maintain a specified constant power output (trimmed rotor). Ref. 10 presented a numerical procedure for the determination of the desired angle and the computational results as applied to the MOD-0 machine were plotted and given in Fig. 4. The total setting angle, θ_B, of an arbitrary blade section at station x is given by (refer to Fig. 1):

$$\theta_B(x,V_o)=\theta_s(V_o)-\theta_o(x) \tag{19a}$$

where θ_o is the initial built-in twist of the blade. The following exponential function is interpolated and found to be a best-fit representation of most of the data given in Ref. [9]:

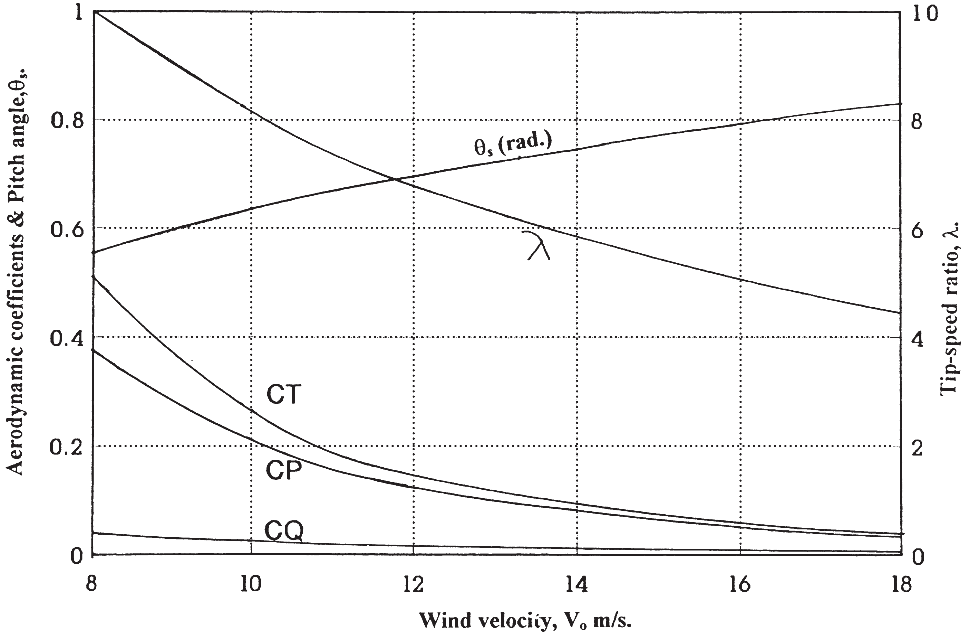

Fig. 4. Pitch angle and aerodynamic coefficients of a trimmed rotor.

$$\theta_o(x)=35.0138[1.0-e^{-3.362x}] \text{ (in degrees)} \tag{19b}$$

The refined blade chord distribution of the actual MOD-0 design is defined by the following dimensionless expressions:

$$\begin{aligned} &C(x)=0.036 && 0.0\leq x\leq 0.2 \\ &C(x)=0.042-0.03x && 0.2\leq x\leq 1.0 \end{aligned} \tag{19c}$$

Using the developed transcendental Eq. (15), Fig. 5 depicts the variation of the angle of attack, α, along the blade span for wind speeds of 8, 10, 12 and 16 m/s. All fall within the operating range of the MOD-0 turbine model. In each case the pitch angle, θ_s, is adjusted by the pitch-control mechanism so that the rotor develops its rated shaft power of about 133 kw (i.e. trimmed rotor). It is observed that α is well-behaved and continuous function in the radial distance x along the blade span. Convergence to a unique solution has been achieved in a minimum time of computation. The present method of analysis eliminates the possibility of having multiple solutions, which may occur when using the traditional iterative procedure [4]. It is to be noticed that a feasible solution of Eq. (15) must ensure positive values of the induction factors. The axial induction factor should not exceed, in any case, a value of 0.5. Fig. 4 also shows the variation of the calculated aerodynamic coefficients with wind velocity.

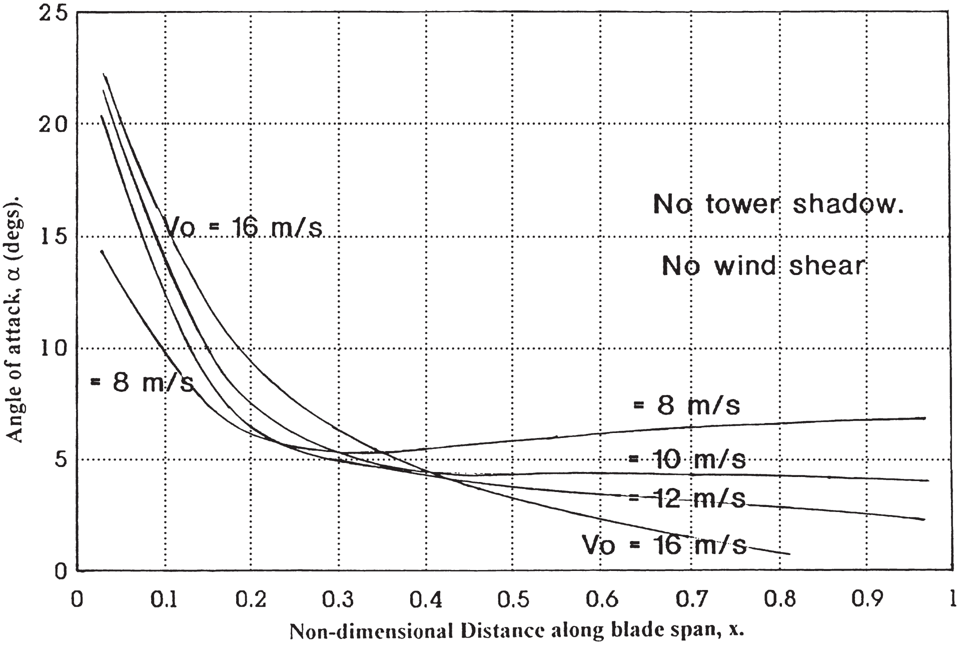

Fig. 5. Variation of angle of attack along blade span for different wind velocities. Trimmed rotor condition (constant power).

5. Conclusions

A new formulation of performance–prediction equations of horizontal-axis wind turbines has been presented. These equations were successfully applied to an existing machine; namely, the ERDA NASA MOD-0, producing 100 kw electrical power. The optimum aerodynamic blade geometry as well as the trimmed-rotor solutions were obtained and investigated in detail. Comparisons with the traditional methods of analysis have shown that the present approach is computationally efficient and ensure continuity of the angle of attack along blade span. This, in turn, guarantees convergence and uniqueness of the attained solutions from the newly developed transcendental equation. Furthermore, it has been indicated that the optimum blade configuration can be adequately determined from an exact trigonometric function method, which is based on Glauert's optimum conditions. In conclusion, the proposed method of analysis eliminates much of the numerical efforts as required by other iterative procedures. Finally, problems that remain are the inclusion of tower shadow and wind shear effects as well as the influence of the choice of the type of airfoil section on the overall aerodynamic efficiency. These problems will be hopefully reported in future studies.

Appendix A

It is convenient to deal with dimensionless quantities so that the analysis can be valid for arbitrary blade configurations and rotor sizes. The various non-dimensional quantities are defined in Table 1. The same symbols that define the actual parameters will be reused to define their corresponding non-dimensional quantities in order to avoid having more subscripts and symbols in the manuscript. For example, the notation $V_r \leftarrow V_r/\Omega R$, as defined in Table 1, means that the dimensionless resultant velocity is equal to its dimensional value divided by the tangential velocity at the blade tip.

Applying blade-element theory, the thrust coefficient can be calculated from:

$$C_T = \frac{T_{aero}}{\frac{1}{2}\rho V_o^2 A_D} = \frac{\int_{r_H}^{R} \frac{1}{2}\rho V_r^2 C C_n N_B dr}{\frac{1}{2}\rho V_o^2 \pi (R^2 - r_H^2)} \tag{A1}$$

Substituting for $r = r_H + x$ ($dr = dx$) and nondimensionalizing the various parameters of the integrand

$$C_T = \frac{\int_0^1 \left(\frac{V_r}{\Omega R}\right)^2 \cdot \left(\frac{C}{2R}\right) \cdot 2R \cdot C_n \cdot N_B \cdot R \left(\frac{L}{R}\right) . d\left(\frac{x}{L}\right)}{\left(\frac{V_o}{\Omega R}\right)^2 \cdot \pi R^2 \cdot \left[1 - \left(\frac{r_H}{R}\right)^2\right]} \tag{A2}$$

Using the notations defined in Table 1, one must get the same form of Eq. (17a), i.e.

$$C_T = \frac{2N_B L \int_0^1 V_r^2 C C_n dx}{\pi V_o^2 (1 - r_H^2)} \tag{A3}$$

Similar derivations shall lead to Eq. (17b). Eq. (17c) is derivable from the definition of the power coefficient.

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